

Chapter 1

Section 1.6

Equations Involving Square Roots

In general, it is easier to work with an equation not containing square roots. This is why we use the formula $a^2 + b^2 = c^2$ instead of $\sqrt{a^2 + b^2} = c$. Therefore, in most instances, we will try squaring both sides of an equation involving square roots. **Hint:** It is usually easier to isolate the square root first.

(1) By isolating the square root and squaring both sides, solve $\sqrt{x} + 12 = x$.

(2) Solve $w = \frac{\sqrt{1-3w}}{2}$.

(3) By squaring each side twice, solve $\sqrt{n+4} + \sqrt{n-1} = 5$.

Equations of Quadratic Type

Def: An **equation of quadratic type** is an equation of the form $au^2 + bu + c = 0$ where u is an algebraic expression.

Q: Is a quadratic equation an example of an equation of quadratic type?

Exercises

(1) Solve the following equation of quadratic type $x^4 - 14x^2 + 45 = 0$.

(2) Solve the equation $(x^2 + x)^2 - 8(x^2 + x) + 12 = 0$.

(3) Solve this equation of quadratic type $x^{2/3} - 9x^{1/3} + 8 = 0$.

Equations with Rational Exponents

Solving an equation with rational exponents involves a similar method to solving an equation with square roots (since a square root is an exponent of $1/2$ which is definitely rational). You first want to isolate your term with rational exponent and then raise both sides to the reciprocal of the rational exponent. **Hint:** Remember our exponent rules, $x^{a/b} = (x^{1/b})^a = (x^a)^{1/b}$.

(1) Solve the equation $x^{4/3} = 625$.

(2) Solve the equation $(y - 2)^{-5/2} = 32$.

(3) If you had troubles with (3) of the previous section, now is the time to go back and finish it.

Q: When dealing with rational equations $x^{a/b} = c$ with $a, b, c \in \mathbf{R}$ and $b \neq 0$ when will your answer require a \pm symbol?

Equations Involving Absolute Value

We have already discussed methods for solving simple absolute value equations. When you have an equation of the form $|u| = |v|$ where u and v are algebraic expressions you just need to remember that this is equivalent to saying u and v are either equal or opposite.

(1) Solve the equation $|x^2 - 6| = 5x$.

(2) Solve the equation $|x^2 - 2x| = |3x - 6|$.
